

Fibonacci numbers in phyllotaxis : a simple model

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A simple model is presented which explains the occurrence of high order Fibonacci number parastichies in asteraceae flowers by two distinct steps. First low order parastichies result from the fact that a new floret, at its appearance is repelled by two former ones, then, in order to accommodate for the increase of the radius, parastichies numbers have to evolve and can do it only by applying the Fibonacci recurrence formula.

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The beautiful spirals observed on sunflowers or daisies have puzzled scientists since it was realized that the numbers of these spirals belonged to the Fibonacci sequence. The basic principle of phyllotaxis has been formulated as early as 1868 by Hofmeister [1] and states that each new structure (leaf, floret, seed, etc.) appears "opposite" to the previous one - or ones. For instance a new leaf often appears opposite to the previous one, which leads to an alternate pattern. When however the new leaf feels the presence, with a smaller amplitude, of the next previous one then the resulting pattern is starry as depicted in fig 1. If leaves appear in pairs, opposite to each other (de-

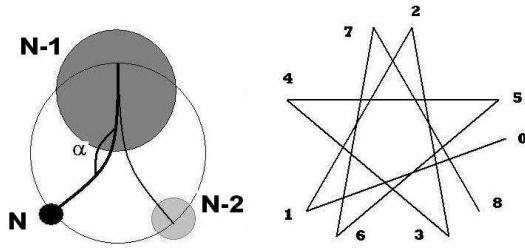


FIG. 1: The new structure N is repelled by its predecessor N-1 and somewhat less by the former one N-2. It appears at an angle α from its predecessor with α between 135° and 144° . At steady state, a starry pattern is formed where each structure is close to its 5th and 8th predecessors and successors.

cussate pattern), each pair is perpendicular to the previous one. It is worth noting that alternate and spiral patterns can coexist in the same plant and that a decussate pattern can turn into a spiral one on a stem: the pattern is sensitive to local influences or noise. In many observed cases and in simulations with a wide range of parameters the angle of the starry pattern lies between $135^\circ = 360^\circ \times 3/8$ and $144^\circ = 360^\circ \times 2/5$ which means that the "structure" of order n will be close to those of order $n \pm 8$ and $n \pm 5$. If the growth is axial this leads to the commonly observed pattern of crossing helices of order 5 and 8. A pineapple is a typical example. If the growth is radial, spirals (in place of helices) of higher

order appear. These orders are not random but belong to the Fibonacci sequence : this sequence is obtained by applying the recurrence equation $F(n+2)=F(n+1)+F(n)$, starting with $F(1)=F(2)=1$. It starts as 1 1 2 3 5 8 13 21 34 55 ... Several investigators [2, 3] have expanded the principle into very elaborate models, considering that the appearance of a new leaf is conditioned by a repulsive potential created by all the previous ones, writing down explicit forms for the model potential, the growth law and the conditions of appearance and computing or simulating the solutions. Subtle changes in any parameter lead to subtle changes in the star angle which would lead to different parastichies. An implicit hypothesis of such computations is that the potential propagates in a perfectly homogeneous medium. A growing plant does not fulfil this condition and the noise due to its heterogeneity would blur such small effects. A sight range of 2 is enough anyway. More involved mechanisms have also been proposed [4].

In the case of radial growth, however a fact which has been apparently overlooked until now is that the parastichies order cannot keep constant along the radius of the flower. The example of a sunflower is shown in fig 2. Let us consider the point at the intersection of the 'o' spiral (parastichy of order 21) and the 'x' spiral (order 34). The next seeds along the o spiral are farther and farther away and no more in contact with each other. The obvious clockwise spiral for the eye becomes the 's' one and interactions between neighbours, if any, occur along it. Computing the new spiral order can be done by looking at fig 3. The appearance of successive seeds along a spiral of order n are separated by $n-1$ others in the other spirals and their rank differ by n . Fig 3 outlines the spiral crossing at seed of rank N and shows the rank of the neighbouring seeds. When the seed of rank $N-21$ is pulled far from the seed N , the nearest neighbour of the latter becomes the predecessor of the former in its order-34 spiral, which is precisely the application of the Fibonacci recurrence formula. The occurrence of high order Fibonacci number parastichies can therefore be explained simply by two distinct steps: The place where each new floret appears is determined by a repulsion from the two previous ones. This induces a starry distribution where a floret is very close to its 5th and 8th predecessors and successors, leading to parastichies of order 5 and 8. In case of radial growth, the spiral with lower order is

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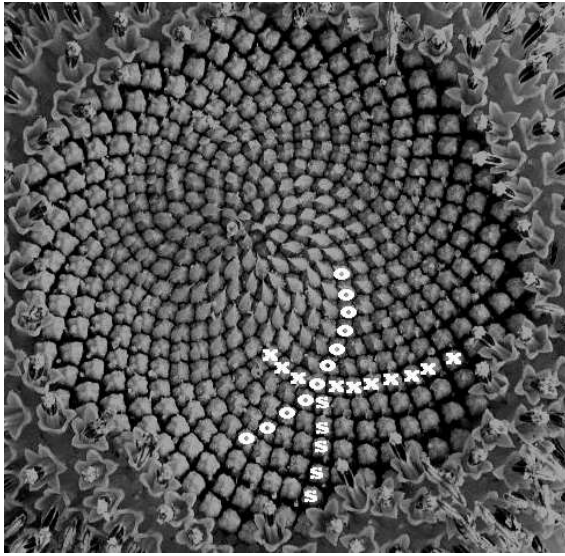


FIG. 2: The spiral of order 21(o) of this sunflower is stretched when the distance from the centre increases and the clockwise next neighbour switches to the s spiral.

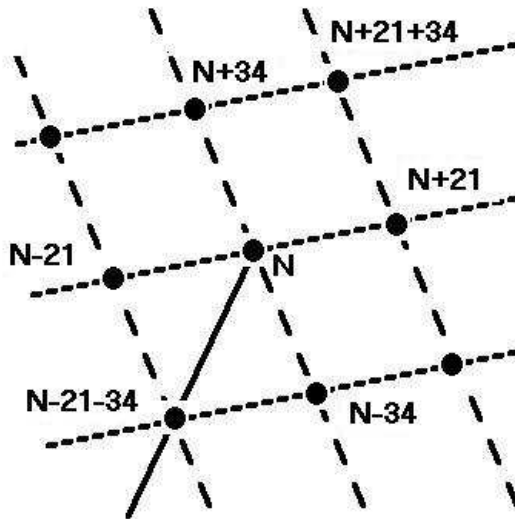


FIG. 3: Scheme of the crossing of spirals of order 21 and 34. When the N-21 point is pulled to the left, the nearest neighbour of the N point in the lower left direction becomes N-21-34. The new spiral order is obtained by following the Fibonacci recurrence formula.

stretched and becomes discontinuous and is replaced by a new one whose order is the sum of the previous ones. Parastichies obeying Lucas sequence have been observed occasionally. This sequence obeys the same recurrence relation but a different couple of starting values. The mechanism of the repulsion has still to be understood. It is believed to involve auxin through complicated mechanisms [5, 6]. It would be also interesting to investigate whether some local interaction stabilises the spiral order, in contrast with some cactuses where a vertical pattern is produced

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